Synopsis on the proposed title

Mathematical studies of ecological and eco-epidemiological models with fear effect

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Mainul Hossain
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under the guidance of

Dr. Nikhil Pal
and
Dr. Tridip Sardar

Department of Mathematics
Visva-Bharati
Santiniketan, West Bengal
India, 731235

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Population biology is an interesting research area to the theoretical ecologists. Interactions among organisms and their environment and the evolution of species are the most important biological processes in ecology and population biology [1, 2]. Mathematical modelling is a powerful tool for investigating the aforementioned biological processes, that is why different types of models have been developed and studied[3, 4]. From the mathematical point of view, we can classify them as deterministic models that include models governed by the ordinary differential equations[5, 4], partial differential equations[3, 2, 6], fractional differential equations [7, 8] and stochastic models[9], models with time delays[10], models with state feedback control[11] and network model[12], etc. Therefore, different types of mathematical models are developed (ODE, PDE, DDE, SDE, etc.) to investigate different biological processes.

Historically, the first model in the population biology was formulated by Malthus [13], and later on, modified for the more realistic situation by Verhulst [14]. Lotka and Volterra independently explained the origin of the cycle in biological populations [15, 16]. The Lotka-Volterra model was subsequently modified by many researchers[17, 18, 19]. Earlier studies on ecological modelling mainly focused on the local stability and persistence [20, 21], whereas recent studies show a trend in exploring broad dynamical behaviours, for example, bi-stability, bifurcation, and chaos [18]. Recent researchers have modified the simple ecological models by considering different biological factors, such as cyclic interaction, Allee effects, disease, alternative food, feeding switching, inducible defence, migration, incubation delay, gestation delay, omnivory etc. and extensively investigated the local and global dynamics of the system and explored the impact of such biological factors on the system stability dynamics.

Eco-epidemiology is a current research area where the effect of epidemics has been investigated on ecological models [22, 23, 24, 25, 26]. It is well established that the disease in a system is an important factor in regulating the population sizes. After the pioneering work of Kermack and McKendrick [27] on SIR (Susceptible-Infected-Recovered) model, various epidemic models (SIR, SIRS, SIS, SEIR, etc.) have been established and investigated extensively, which leads to the progress in the studies of disease control and prevention. In modelling the transmission of an infectious disease, a common model structure involves dividing the population into different groups. For example, in an SIR(Susceptible-Infected-Recovered) model, structure involves dividing the populations into susceptible, infectious and recovered individuals. A susceptible (S) individual becomes infected (I) by the contact with an infected individual and the process is interpreted as a catalytic reaction. An infected individual may be recovered (R) from the disease forever by permanent immunization and the process described as a spontaneous reaction. However, if infected individuals don't acquire permanent immu-

nization, it becomes the susceptible-infected-recovered-susceptible (SIRS) model. If an infected individual spontaneously recovers from the disease becoming again susceptible, it becomes susceptible-infected-susceptible (SIS) model. There may be other categories depending on the clinical and epidemiological properties of the disease. For example, in an SIR model, if there is a significant incubation period during which the individual has been infected but is not yet infectious, then it becomes susceptible-exposed-infected-recovered(SEIR) model. Many eco-epidemiological models considered prey-predator interactions with an infection in prey species [23, 28]. Few works are done by considering the infection in predator population [24, 29].

In the study of ecosystems, predator-prey interaction plays the most important role in determining the future population size through mathematical modelling. There are two different approaches to capture the predator-prey interactions. In the first kind of approach (also known as the direct approach) predators kill prey [30]. In the second type of approach(indirect approach), the fear of predation risk(i.e. the fear of being killed) puts the prey individuals into a psychological stress [31], which in turn affects their behaviour. The first approach is a property of the predator population whereas the second approach is a property of the prey populations. Till date, numerous researchers have come up with various functional responses describing different kinds of interaction among predator and prey individuals. In earlier studies of predator-prey models, it was assumed that the impact of the predators on the prey species lies just in predation. But in many cases, this traditional idea fails to explain what happens in reality. A recent emerging view is that indirect effect (fear effect) is powerful enough to change the dynamics of a predator-prey system [32, 33, 34, 35]. Empirical studies have recently shown that the fear effect can be large, sometimes substantially larger than direct effects[36, 37, 38, 39, 40, 41, 42, 43].Without consideration of the indirect effects of predation, it is likely that decreased reproduction would be mistaken for bottom-up limitation by resources or be attributed to processes other than predation, particularly when risk effects reduce reproduction. The study of Pangle et al. [44] on the fear effect of spiny water fleas(*Bythotrephes longimanus*) on three different zooplankton species showed that the fear effect is more than seven times larger than the direct effect in terms of population growth. Perceived predation risk can alter the behaviours of the scared prey like foraging. For example, mule deer become more vigilant and don't go out to search for foods widely when they sense predation risk of mountain lions [45]. Many prey individuals move to safer habitats having lower predation risk. If the habitat has less food than the required level, it costs energy loss of prey species. Wirsing et al. [46] observed that the dugongs (a medium-sized marine mammal) underused edge (safe) microhabitat when sharks were scarce, overused it when sharks were common, and responded to daily changes in shark abundance in a threat-sensitive fashion, showing the greatest preference for edges when shark abundance was highest. Many researchers experimentally observed that fear of predation risk can cause lower birth rate in prey population. Zanette et al.[32] carried out an experiment on song-sparrows over a whole breeding season by playing sounds of their predators to some of them and non-predator sounds to the rest. They also protected their nest to eliminate any chance

of direct killing. They observed about 40% reduction in the number of offspring of the song sparrows due to fear alone. The reason behind this fall was the decrease in the number of eggs laid by the female song-sparrow, decrease in the number of hatched eggs and the survival of the offspring which refers to the death of the nestlings. Due to fear parent birds could feed their offsprings less than what they need. This led to health problems like lightweight and made them less survivable. Suraci et al. [34] experimentally manipulate the fear of large carnivore on mesocarnivores.
They showed that fear affects mesocarnivore's foraging behaviour (lower by 66%) and benefit to the mesocarnivore's prey. Fear of large carnivores caused mesocarnivores to spend less time in the foraging zone and increase their vigilance, which lowers their food intake and reduce their chance of being killed. Such a complex strategy increases their survival probability and plays a central role in the evolutionary process. Animals like elk, dugongs, snowshoe-hares were observed to change their reproductive physiology as per predation risk [36]. Many researchers have also observed that fear effect causes lower reproduction rate in some species and makes the juveniles less survivable [32, 34, 47, 36, 48]. Some researchers[41, 43] observed the effect of fear even in the absence of direct killing.

Functional responses represent the direct effect(predation). As the effect of fear on environment became clear, many researchers started to consider fear effect in ecological systems. Although the literature on fear effect dates back to about a century, the very first mathematical model was first proposed by Wang et al.[35] in 2016. They proposed a two-dimensional predator-prey model incorporating fear effect in the growth term of the prey. They observed that fear can stabilize the system by excluding periodic solutions. The most important result they explored was that for a suitable combination of parameter values, the impact of fear can change the stability property of the limitcycle oscillation. Fear can change population oscillation from supercritical Hopf bifurcation to subcritical Hopf bifurcation which was absent in previous classical predator-prey models. The following models are proposed by Wang et al.[35].

Model 1:

$$\begin{array}{rcl} \frac{\mathrm{d}u(t)}{\mathrm{d}t} & = & \frac{ru(t)}{1+kv(t)} - du(t) - au^2(t) - pu(t)v(t), \\ \frac{\mathrm{d}v(t)}{\mathrm{d}t} & = & cpu(t)v(t) - mv(t), \end{array} \tag{1}$$

Model 2:

$$\begin{array}{rcl} \frac{\mathrm{d}u(t)}{\mathrm{d}t} & = & \frac{ru(t)}{1+kv(t)} - du(t) - au^2(t) - \frac{pu(t)v(t)}{1+qu(t)}, \\ \frac{\mathrm{d}v(t)}{\mathrm{d}t} & = & \frac{cpu(t)v(t)}{1+qu(t)} - mv(t), \end{array} \tag{2}$$

where $u(t)$ and $v(t)$ be the prey and predator populations, respectively. r is the birth rate of prey, d is the natural death rate of prey, a is the death rate of prey due to intra-specific competition, p is the maximum predation rate of the predator, q is the constant related to half-saturation constant, c represents the conversion efficiency of the predator, m is the natural mortality rate of the predator, k is the fear parameter. It is to be noted that due to fear of predation risk, the reproduction rate of prey reduces.

Very recently, Panday et al. [49] investigated a three-species food chain model, where the growth rate of middle predator is reduced due to the cost of fear of the top predator, and the growth rate of prey is reduced due to the cost of fear of the middle predator. They observed that fear can stabilize a chaotic system.

The model proposed by Panday et al.[49] is

$$\begin{aligned} \frac{\mathrm{d}X}{\mathrm{d}T} &= \frac{R_0}{1+K_1Y}X(1-\frac{X}{K_0}) - \frac{C_1A_1XY}{B_1+X}, \\ \frac{\mathrm{d}Y}{\mathrm{d}T} &= \frac{A_1XY}{B_1+X}\frac{1}{1+K_2Z} - \frac{A_2YZ}{B_2+Y} - D_1Y, \\ \frac{\mathrm{d}Z}{\mathrm{d}T} &= \frac{C_2A_2YZ}{B_2+Y} - D_2Z. \end{aligned} \tag{3}$$

Here, X, Y and Z represent the densities of prey, middle predator, and top predator population, respectively. In the absence of predation, the prey population grows logistically with an intrinsic growth rate R_0 and carrying capacity K_0. A_1 and A_2 are the maximum predation rates of the middle predator on prey and the top predator on the middle predator, respectively. B_1 and B_2 are the half-saturation constants of the middle predator and top predator, respectively. ${C_1}^{-1}$, C_2 represent the conversion efficiencies of middle predator and the top predator, respectively. D_1 and D_2 are the natural mortality rates of the middle predator and the top predator, respectively. They incorporated the cost of fear in the three-species food chain model. They assumed that the growth rate of the middle predator is reduced in the presence of the top predator, i.e., the modified growth rate of the middle predator is $\frac{A_1}{1+K_2Z}$ - a monotonic decreasing function of both K_2 and Z. Similarly, the modified intrinsic growth rate of prey is $\frac{R_0}{1+K_1Y}$ - a monotonic decreasing function of both K_1 and Y. K_1 and K_2 are the fear parameters of prey and middle predator, respectively.

So, there is a huge scope to study the dynamics of ecological and eco-epidemiological models under the influence of fear effect. The aim of this research is to develop some mathematical models with the fear effect and investigate the models rigorously by using both mathematical and numerical techniques. I hope the successful completion of this research may generate a considerable interest among the researchers in India.

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